Ordinary differential equation

Solve the following ordinary differential equation

$$-\left(x^2y+y\right)y'+x=0$$

Solution

$$-\left(x^2y+y\right)y'+x=0$$

Reescribimos

$$-y(x^2+1)y'+x=0$$

$$-y(x^2+1)y'=-x$$

$$y(x^2+1)y'=x$$

$$yy'=\frac{x}{x^2+1}$$

Ahora, separando variables:

$$y\,dy=\frac{x}{x^2+1}\,dx$$

Integrando ambos lados:

$$\int y\,dy=\int\frac{x}{x^2+1}\,dx$$

Calculamos las integrales:
Lado izquierdo:

$$\int y\,dy=\frac{1}{2}y^2+C_1$$

Lado derecho:
Utilizamos sustitución $u=x^2+1$, entonces $du=2x\,dx$:

$$\int\frac{x}{x^2+1}\,dx=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln|x^2+1|+C_2$$

Igualando las integrales:

$$\frac{1}{2}y^2=\frac{1}{2}\ln(x^2+1)+C$$

$$y^2=\ln(x^2+1)+C$$

Finalmente, despejamos y:

$$y=\pm\sqrt{\ln(x^2+1)+C}$$